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NOTES ON ANALYTICAL TECHNIQUES IN AERODYNAMIC HEATING PROBLEMS

By

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and
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ABSTRACT. This report is a compilation of several informal notes concerned with a variety of analytical techniques used in aerodynamic heating problems. Two of the notes describe heat flow studies using analog techniques, and the last considers the criteria under which a skin may be termed "thermally thin."

U. S. NAVAL ORDNANCE TEST STATION

China Lake, California

28 April 1961

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FOREWORD

This report is a collection of informal papers by various authors on the applied research effort at this Station in aerodynamic heating. Principal support for the work was received from the Bureau of Naval Weapons Task Assignment RMGA-53-406/216-1/F009-10-001 with some assistance from various development tasks.

The report is released at the working level for informational purposes only. It contains information which is still subject to modification or withdrawal.

LEROY RIGGS
Head, Aeromechanics Division

Released under
the authority of:
F. H. KNEMEYER
Head, Weapons Development Department

INTRODUCTION

A program of applied research in the problem of aerodynamic heating of guided missiles has been in progress as a funded task at the Naval Ordnance Test Station for the past three years. In addition to several relatively long-term major efforts, a number of investigations of smaller scope form a significant portion of this program. In many instances, these latter investigations grew out of or were extensions to work required for a specific development program. In these cases, the results were generally published in local informal reports of very limited distribution, or were submerged in a more extensive report concerning the related development task.

This report is one of several that are being published to provide easier access and wider distribution to the results attained in these investigations. These reports consist of a collection of notes on related topics gleaned from the informal reports which were published for local distribution only.

The notes in this report are concerned with analytical techniques. The methods described utilize analog computers, and criteria are given under which more simplified methods may be profitably employed.

SECTION I. TEMPERATURE CHANGES DURING HEATING OF AN HPAG 5.0 INCH MOTOR: CORRELATION OF EXPERIMENTAL DATA WITH ANALOG COMPUTATIONS

By

C. H. Johnson

INTRODUCTION

A major difficulty in estimating temperature histories inside missile components subjected to unsteady-state aerodynamic heating is in setting up a suitable model and finding accurate physical property data to use in the computations. Deficiencies in thermal property data for engineering materials, not only with respect to accuracy at ambient temperatures but also in regard to temperature dependence, make it impractical to consider any other than principal effects on heat transfer in setting up the analytical model. Nevertheless, it is important for good simulation to include all the essential effects, and laboratory experiments may be needed to determine which effects should be considered in analytical studies. This note illustrates how temperature data on a 5.0-inch HPAG rocket motor served to guide the set-up of the analog computer model. The study was prompted by observations that temperatures in Sidewinder 1C motors, heated in the NOTS radiant heat facility, differed appreciably depending on whether the test point was at the top, side, or bottom of the motor.

THE PROBLEM

The HPAG motor was used as the experimental model in this study because of its similarities to current missile propulsion units. Data on this and other motors subjected to oven heating and cooling tests are given in Ref. 1. The present study is based on a step increase in temperature, 78° to 140°F. This interval was chosen so that available thermal conductivity, specific heat, and thermal expansion data would apply, and the effect of temperature on these properties would be minimized. All motor grain physical properties as listed below were taken from Ref. 2 as typical for double-base propellants, and motor dimensions were taken from Ref. 3.

Component	Material	Density lb/in^3	Specific Heat Btu/lb-°F	Thermal Conductivity Btu/in-sec-°F
Motor Tube	2014 Al Alloy	0.10	0.22	20.6×10^{-4}
Inhibitor	Ethylcellulose	0.04	0.40	2.60×10^{-6}
Propellant	N-4	0.058	0.45	2.78×10^{-6}

In determining temperature histories in this type of problem, it is the usual practice to apply the computed heat transfer (film) coefficients and the external driving temperatures as the forcing conditions which cause temperature build-up in the motor. In this case, however, the measured temperature of the motor tube was used as the forcing function on the grain, thereby isolating the motor from its environment for which characteristics could not be accurately determined in the actual tests.

ANALOG COMPUTER RESULTS

Figure 1 shows the actual temperature histories of the motor wall, the inhibitor-grain interface, and the grain interior near the perforation in curves 1, 2, and 3, respectively. The measurements were made at identically located points on the top and one side of the motor; however, only the grain interface temperatures were strongly position-dependent, accounting for the different curves 2 and 2'. Computed temperature histories at the same positions, assuming grain properties as given in Ref. 2, are shown for zero air-gap by curves 2A and 3A and for constant 0.045-inch air-gap at the motor side by curves 2B and 3B. The latter case corresponds to the maximum allowable air-gap under HPAG motor specifications. These computed curves for extreme conditions show poor agreement with the measured temperatures, and additional computations (not included here) disclosed that neither a reduced constant air-gap nor different grain thermal properties appreciably improved the agreement.

Figure 2, curves 2C and 3C, compares the actual results with the computed temperatures with properties as before, except that a 0.020-inch initial air-gap at the motor side continuously changed until full contact between grain and motor tubes was obtained. The computed values for a grain with increased thermal diffusivity but the same air-gap conditions is shown by curves 2D and 3D. A considerably improved fit with actual data is evident when compared with Fig. 1; however, reduction of the initial air-gap and a smaller increase in thermal diffusivity as shown in curves 2E and 3E of Fig. 3 are required for an adequate match with the experimental values. Further attempts to improve agreement were considered inappropriate since the effects of factors such as unevenness of tube-grain contact and changes in thermal diffusivity of the propellant during heating were not included in the analysis.

The effect of grain heating on grain growth is shown for a typical case in Fig. 4, and the basis for computing growth is described in "Method of Calculating Thermal Expansion" (p.10).

HEAT TRANSFER COEFFICIENTS

The over-all heat transfer coefficient for the heating run, as calculated in Ref. 2, is shown in Fig. 5. In addition, Fig. 5 shows the continuous heat transfer coefficients calculated in this study by two different methods.

One method required a plot of dT_1/dt versus time for the motor tube (to avoid differentiating on the computer) which was used in solving the equation

$$h = \frac{q_{1-2} + \left(dT_1/dt\right) C_1}{T_\infty - T_1}$$

where

h = film coefficient, $Btu/in^2\text{-sec-}°F$

q_{1-2} = heat transfer rate wall-to-grain, $Btu/in^2\text{-sec}$

C_1 = heat capacity of motor wall, $Btu/in^2\text{-}°F$

T_1 = motor wall temperature, °F

T_∞ = oven temperature, °F

The film coefficient (h) is obtained by dividing the instantaneous heat exchange at the motor wall by the difference between the motor wall and the oven temperatures.

The second method required manual adjustment of the film coefficient by a trial-and-error operation so that continuous agreement between actual oven temperature and motor-wall temperature was maintained. Both methods indicate lower film coefficients during the early part of the heating period and support the belief of the Ref. 1 authors that oven variables did cause variations in the film coefficient.

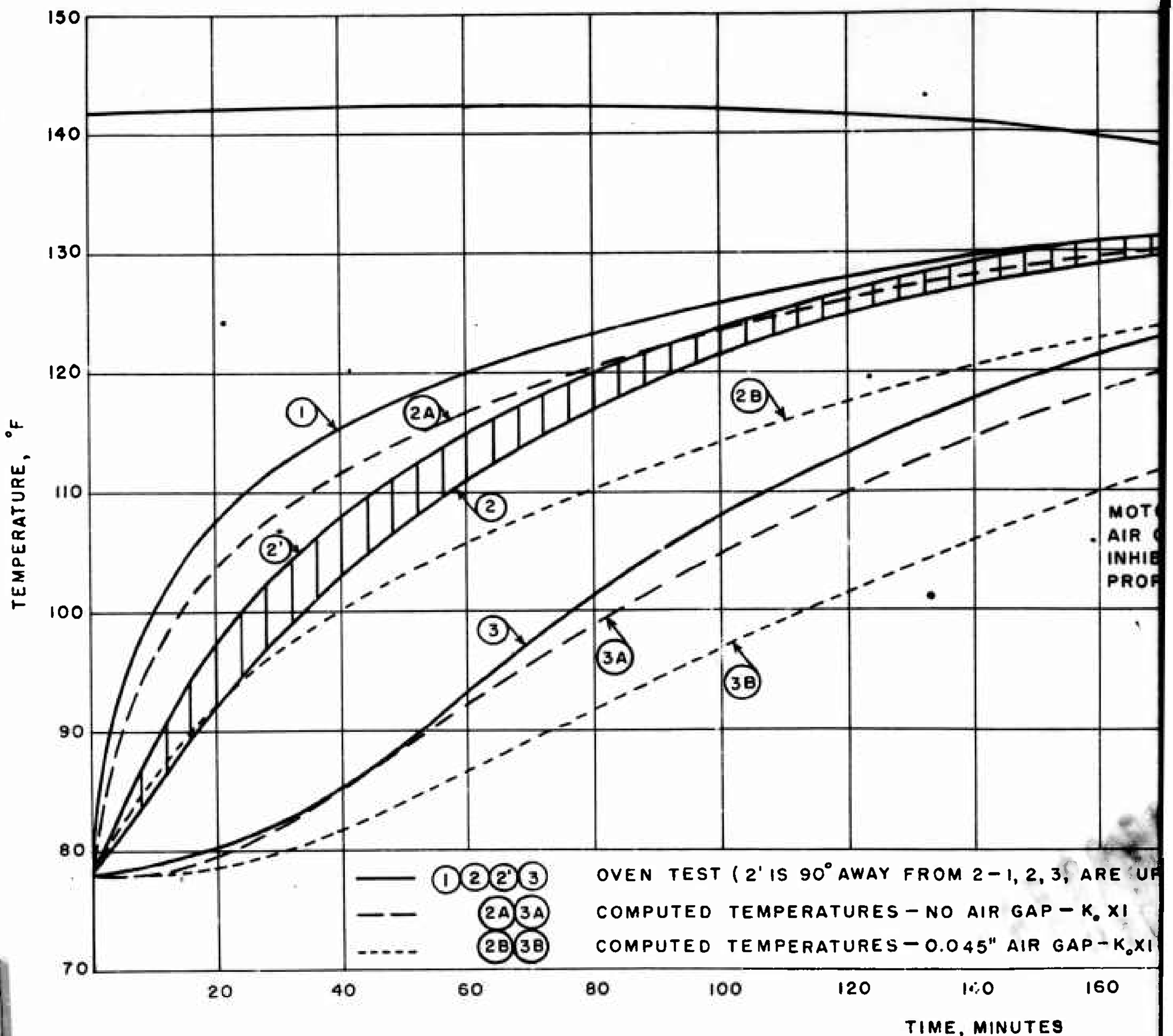

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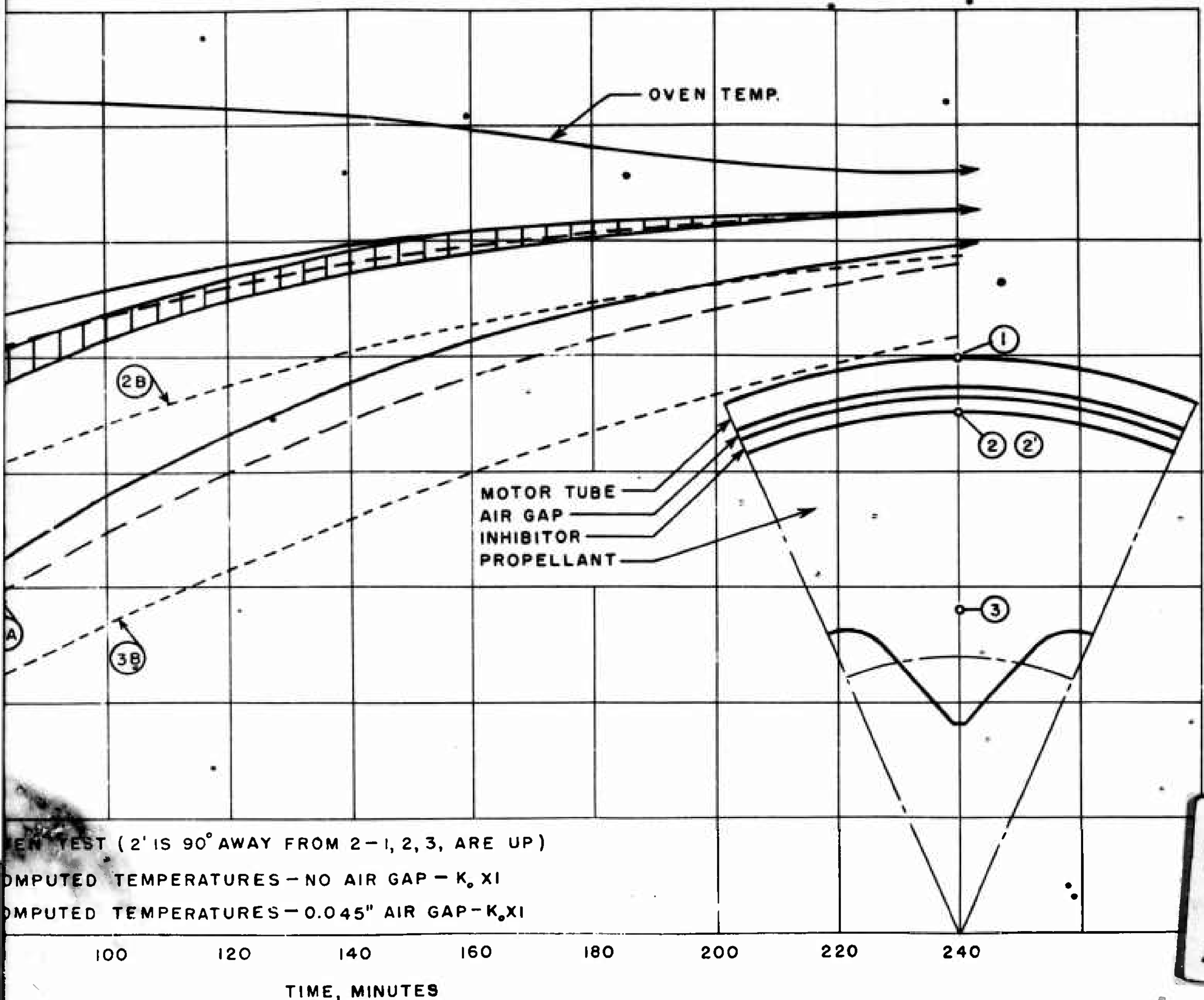
FIG. 1. Heating Curves for the 5.0-Inch HPAG



1. Heating Curves for the 5.0-Inch HPAG Rocket Motor.

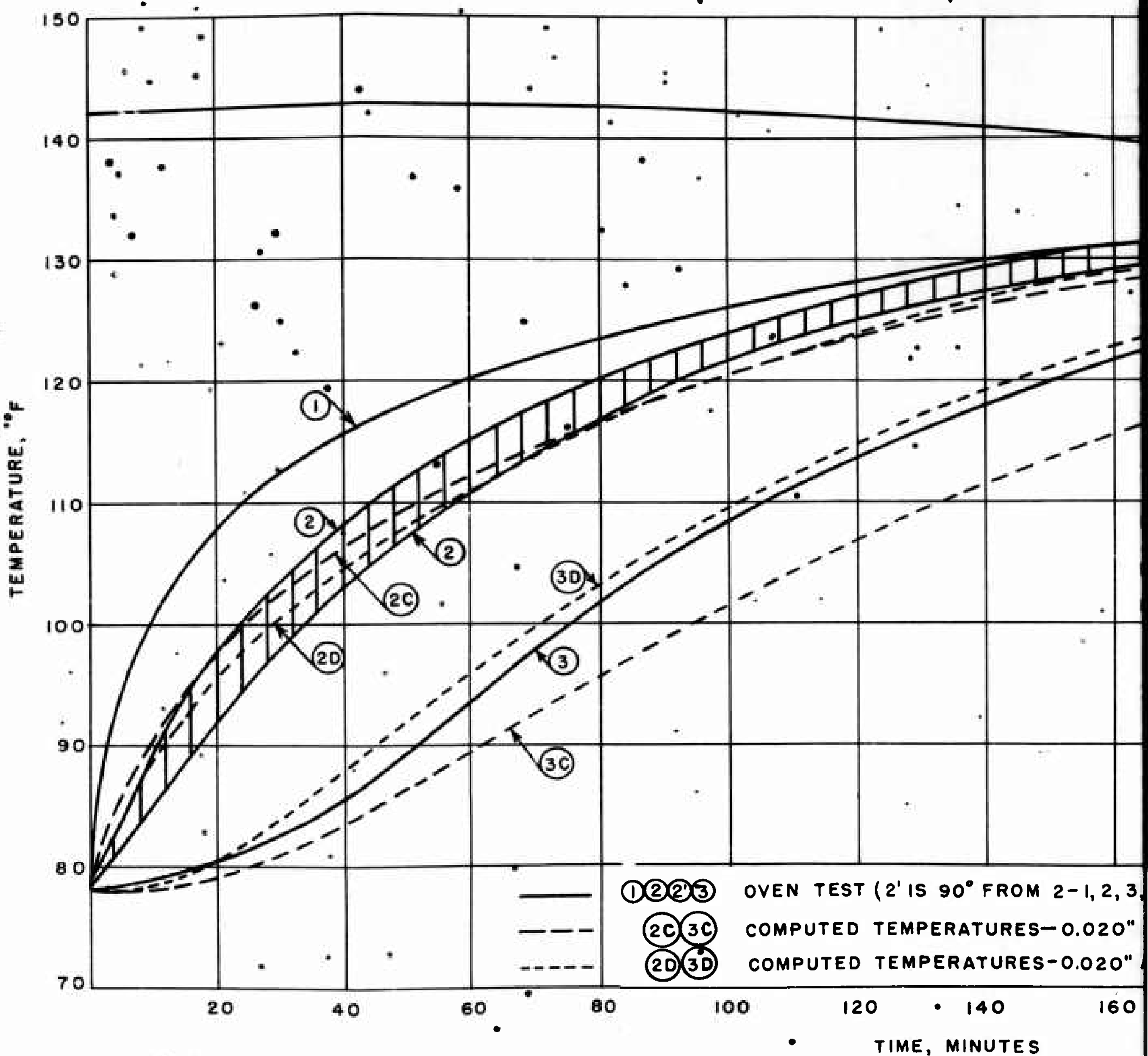
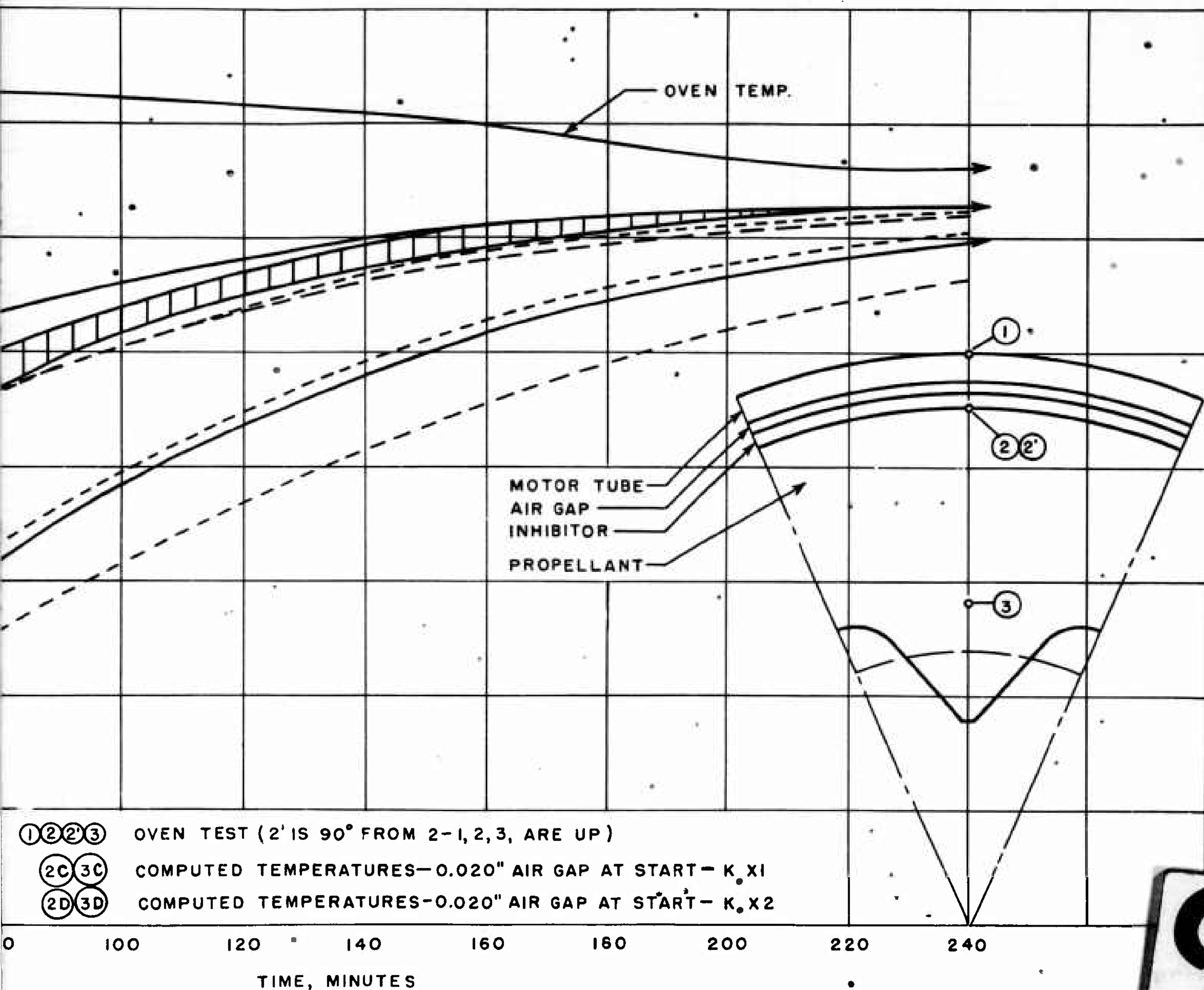


FIG. 2. Heating Curves for the 5.0-Inch HPA



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. 2. Heating Curves for the 5.0-Inch HPAG Rocket Motor.

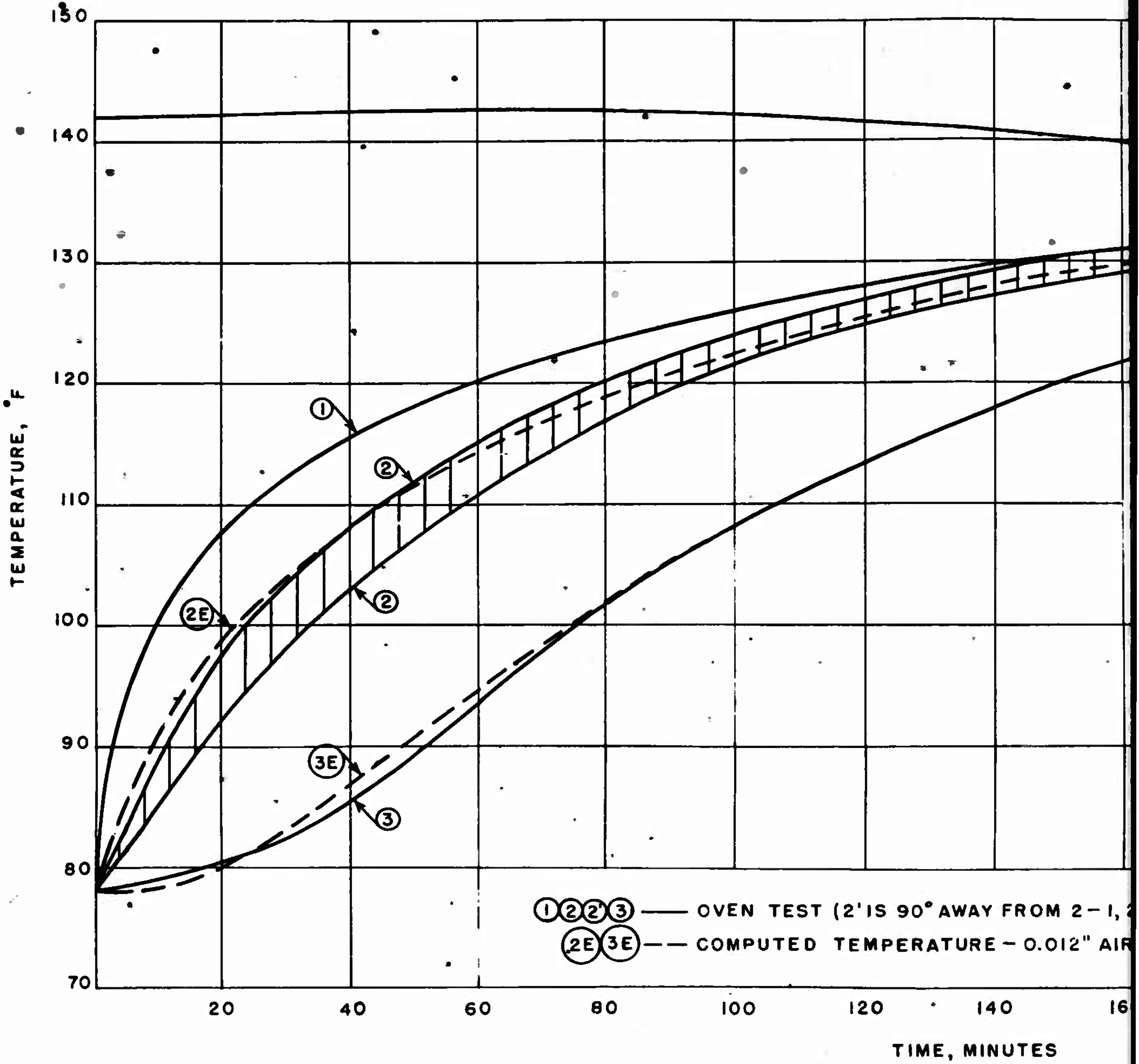
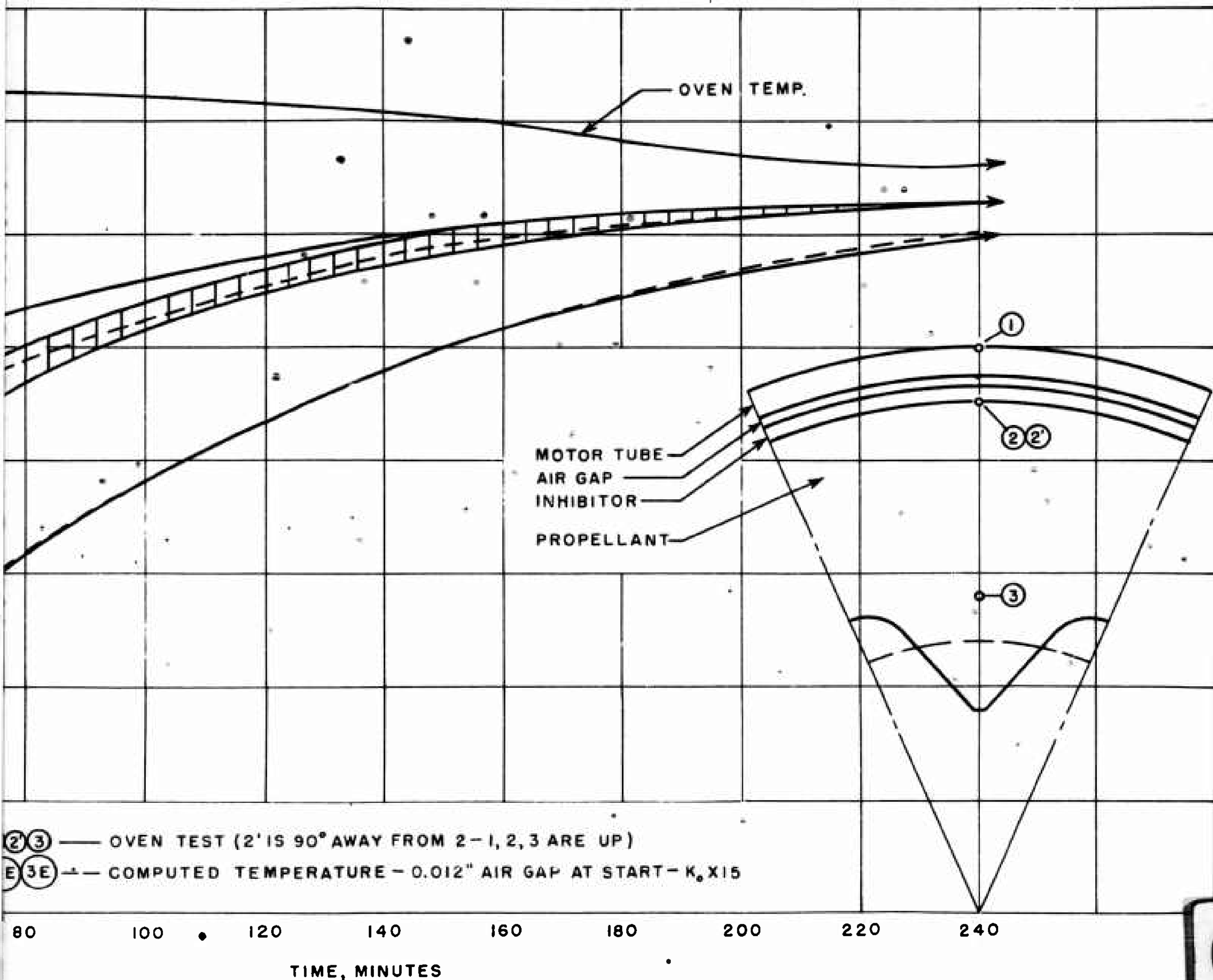


FIG. 3. Heating Curves for the 5.0-Inch HPA



G. 3. Heating Curves for the 5.0-Inch HPAG Rocket Motor.

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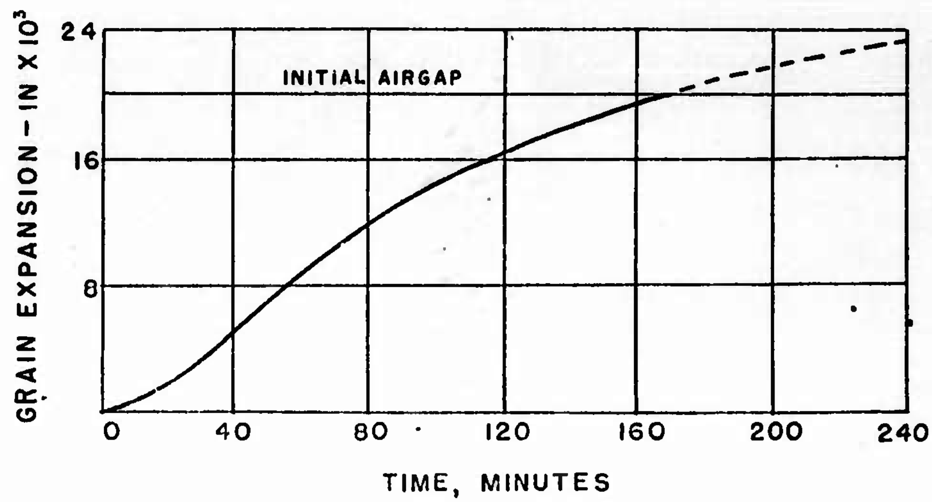


FIG. 4. Computed Grain Growth During Heating Period.

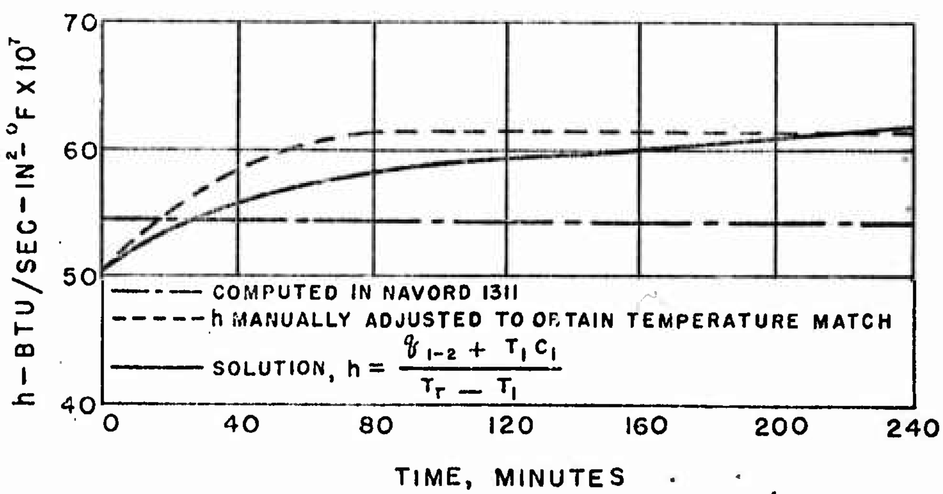


FIG. 5. Computed Heat Transfer Coefficients.

METHOD OF CALCULATING THERMAL EXPANSION OF HPAG MOTOR GRAIN

Basic Assumptions

1. Coefficients of thermal expansion, as taken from Ref. 2, are taken equal for radial and tangential directions.

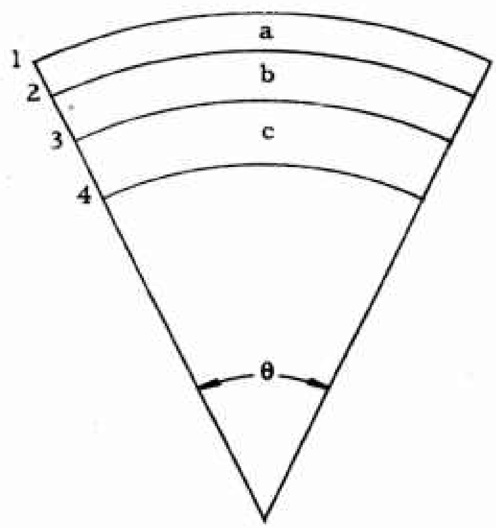
2. Grain perforation is taken as a circular cylinder with all growth non-elastic and non-plastic.

3. Nominal growth of each zone in grain is proportional to temperature increase, and growths are additive radially.

4. All grain deformations are plastic and are additive radially.

Calculations

In the analog computation for this problem, the grain model was set up with seven concentric zones. The following discussion is based on three zones in the interest of simplicity.



The radius of zone (k) boundaries is r_{n+1} and r_n, and the mean radius of zone (k) is

$$r_{mk} = \frac{r_n + r_{n+1}}{2}$$

The mean length of zone (k) segment is $\ell_{mk} = r_{mk}\theta$, and its radial thickness is

$$\delta_k = r_n - r_{n+1}$$

The temperature at mid-thickness of zone (k) is T_{mk}. The perforation expansion is given as

$$\Delta r_4 = r_4 \alpha \Delta T_4$$

and, therefore, the new perforation radius is

$$r_4(1 + \alpha\Delta T_4).$$

The circumferential and radial expansion of zone (c) would be

$$\Delta\ell_{mc} = \ell_{mc}\alpha\Delta T_{mc} = r_{mc}\theta\alpha\Delta T_{mc}$$

$$\Delta\delta_c = \delta_c \alpha\Delta T_{mc}$$

and the new area of zone (c) becomes

$$\begin{aligned} A'_c &= (\ell_{mc} + \Delta\ell_{mc})(\delta_c + \Delta\delta_c) \\ &= r_{mc}\theta\delta_c (1 + \alpha\Delta T_{mc})^2 \\ &\cong r_{mc}\theta\delta_c (1 + 2\alpha\Delta T_{mc}) \end{aligned}$$

since $\alpha^2 (\Delta T_{mc})^2$ is a very small quantity.

This area increase is now constrained to allow only radial growth (θ = constant) built on the zone minor radius; that is,

$$\begin{aligned} \delta'_c &= \frac{A'_c}{\ell_{mc}} = r_{mc}\theta\delta_c \frac{(1 + 2\alpha\Delta T_{mc})}{r_{mc}\theta} \\ &= \delta_c (1 + 2\alpha\Delta T_{mc}) \end{aligned}$$

and the radial growth of zone (c) is then

$$\Delta\delta'_c = 2\delta_c \alpha\Delta T_{mc}$$

It is now assumed that the expansion of the perforation and zone (c) does not appreciably affect computation accuracy for zone (b) using the same method as for zone (c); that is,

$$r'_{mb} \cong r_{mb} + \Delta r_4 + \Delta \delta'_c + \frac{1}{2}\Delta \delta'_b$$

with an error of less than 2 percent at the end of the heating period. Therefore, the radial growth of zone (b) is

$$\Delta \delta'_b = 2\delta_b \alpha \Delta T_{mb}$$

and similarly for zone (a),

$$\Delta \delta'_a = 2\delta_a \alpha \Delta T_{ma}$$

The maximum radius, r'_1, of the grain at time, τ, is accordingly:

$$r'_{1(\tau)} = r_4(1 + \alpha \Delta T_4) + (\delta_a + \delta_b + \delta_c) + (\Delta \delta'_a + \Delta \delta'_b + \Delta \delta'_c)$$

$$= r_1 + r_4 \alpha \Delta T_4 + 2\alpha(\delta_a \Delta T_{ma} + \delta_b \Delta T_{mb} + \delta_c \Delta T_{mc}).$$

ACKNOWLEDGMENT

The author wishes to acknowledge the assistance of the Analog Studies Branch of the Research Department who produced the analog curves used in this report.

SECTION II. DETERMINATION OF WALL TEMPERATURE-TIME FUNCTIONS USING AN ANALOG COMPUTER

By

William H. Smith

EDITORIAL PREFACE

Work done subsequent to the preparation of this paper has shown that one of the simplifying assumptions--that of one-dimensional flow--may lead to significant inaccuracies in the cases chosen as examples. Because the heat flux changes appreciably with position over a nose cone, lateral conduction may not, in general, be neglected with impunity. The methods described are reasonably valid, however, for cases where the heat flux is constant over a significant area.

GENERAL DISCUSSION

Solution of the unsteady-state heat balance equations for determining nose cone stagnation temperatures may be accomplished using an analog computer. This section discusses an analog computer method which was used for this purpose and as an illustrative example.

Prior to computer evaluation of the stagnation temperature of a surface, certain functions must be obtained. The shape, dimensions, and material of the object of interest should be known since density, thermal conductivity, specific heat, thickness, and possibly other parameters are used as constants in analog computer solutions. The recovery temperature and heat transfer coefficient must be determined as a function of time. Graphs of these functions versus time are necessary in order to insert these transients into the function generators of an analog computer. Methods for calculating recovery temperatures and heat transfer coefficients are given in Ref. 4 and 5 and in an informal technical note.[1] Programs have been prepared for computing these functions using the IBM 610 and 704 digital conputers.

Figures 6 and 7 show heat transfer coefficient and recovery temperature as a function of time for the flight profile chosen for the example. Figures 8 and 9 show the two nose-cone configurations which were evaluated.

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[1] U. S. Naval Ordnance Test Station. An Estimate of the Recovery Temperature and Heat Transfer Coefficient over the Sidewinder 1C Nose at Mach numbers 1.9, 2.5, and 3.0, by L. L. Doig. China Lake, Calif., NOTS, 27 January 1958. (NOTS Technical Note 4061-5), CONFIDENTIAL.

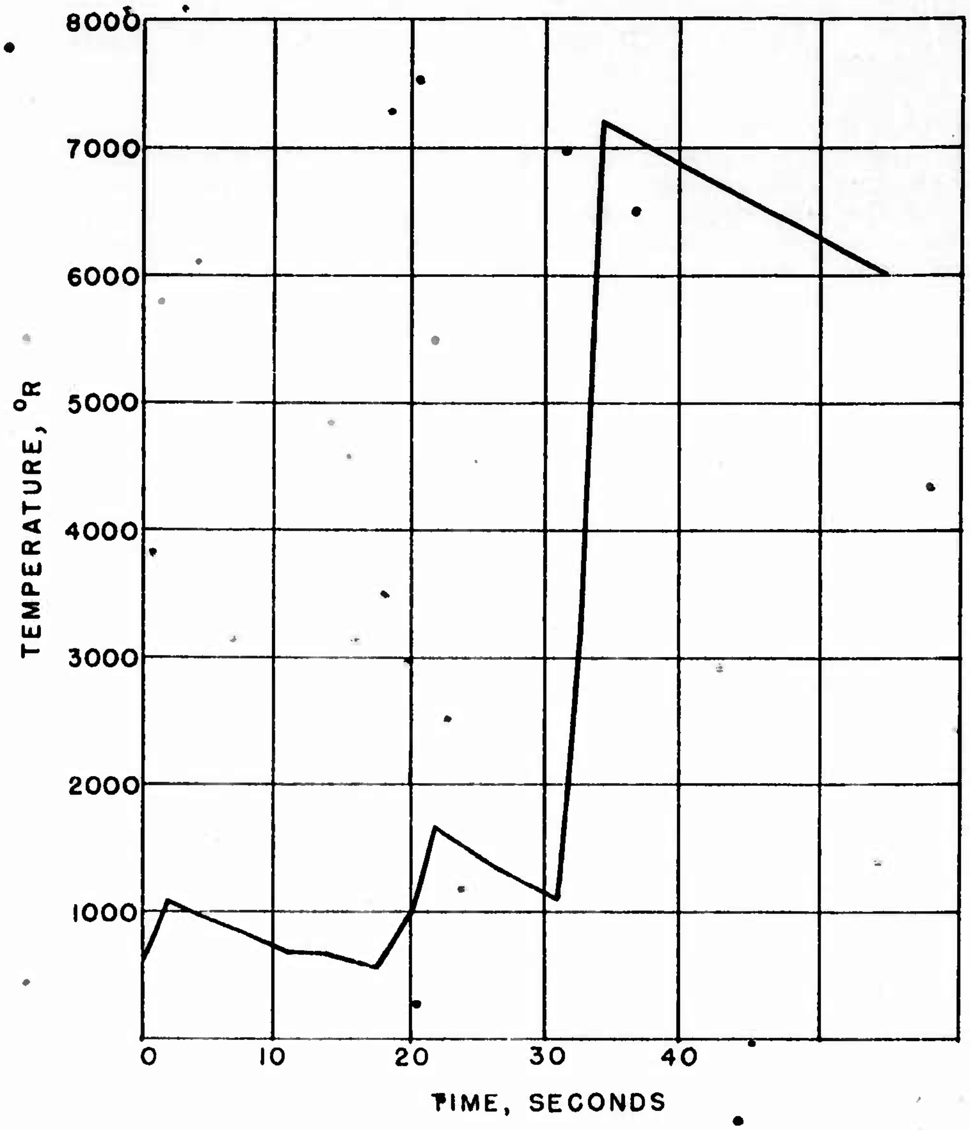

FIG. 6. Recovery Temperature Versus Time.

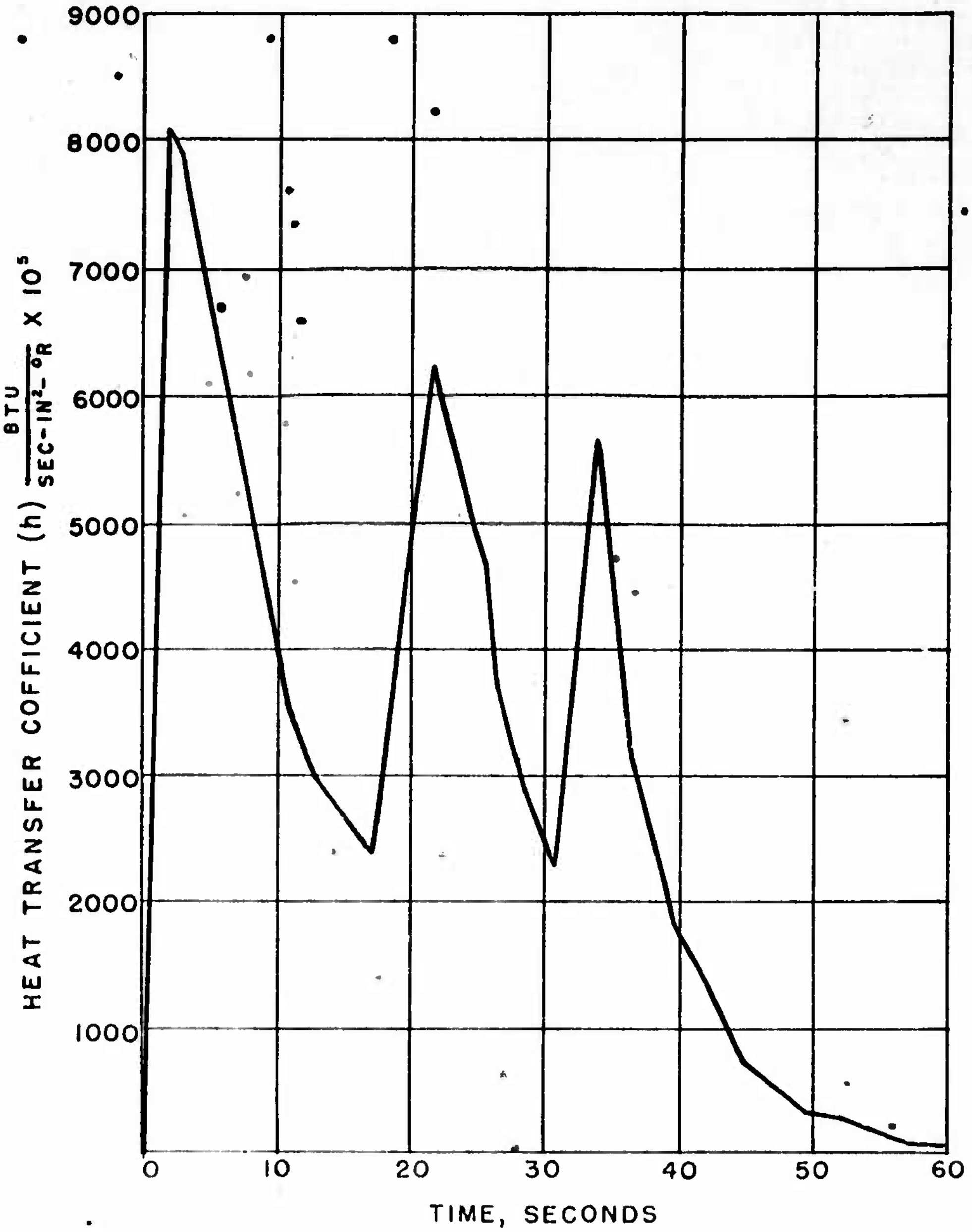

FIG. 7. Heat Transfer Coefficient Versus Time (6.9-inch Diameter Hemisphere).

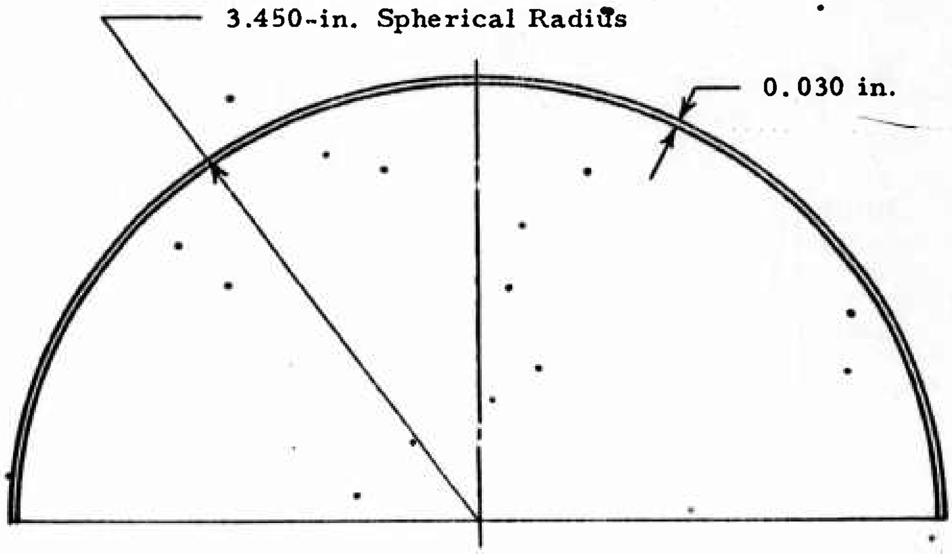

FIG. 8. Thermally Thin Configuration--
Stainless Steel.

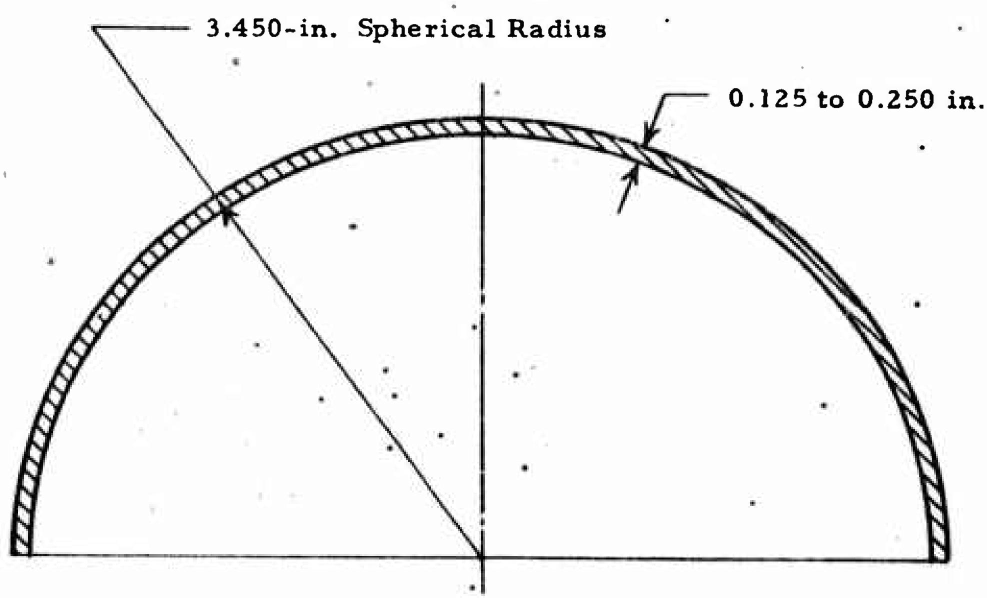

FIG. 9. Thermally Thick Configuration--
Astrolite.

Certain simplifying assumptions were made in obtaining the solution. These include:

1. One-dimensional heat flow

2. Radiation and convective heat transfer from the skin to the interior of the missile are neglected

3. No ablation.

The term "REAC" on the graphs refers to curves obtained from the analog computer solutions. The circuits shown in Fig. 10 and 11 are representative and do not include such things as intermediate amplifiers and scaling factor potentiometers which are needed in actual analog computer solutions. A thorough knowledge of analog computer circuits is needed for the solution of particular problems and a person with such a knowledge should be consulted before any solutions are attempted.

THERMALLY THIN SECTIONS

Sections which have a temperature gradient of 10% or less were considered to be thermally thin in this application. The percent temperature gradient across particular designs may be obtained by using Section IV of this report. The thermally thin design considered is shown in Fig. 8.

The equation used to determine the temperature for thermally thin sections is

$$\frac{dT_w}{dt} = \frac{h}{\overline{w}C\tau}(T_r - T_w) - \frac{\sigma\epsilon}{\overline{w}C} T_w^4 \qquad (1)$$

The computer circuit (Fig. 10) needed to solve this equation is quite simple. Approximately 4 hours were needed to wire the control panel, put the recovery temperature and heat transfer coefficient into the function generators of the Mid-Century Analog Computer, and read out solutions for the problem.

Figure 11 shows the skin temperature as a function of time using results obtained from computer solutions of Eq. 1. The values of emissivity were arbitrarily chosen at 0. 0, 0. 5, and 0. 85.

Several methods of numerical integration are available for hand solution of Eq. 1. These include repetitive calculations, trial and error, and the Runge-Kutta methods, and are fully explained in Ref. 6. When the radiation effect is included, the only one of these methods applicable is the trial-and-error solution. However, the effects of radiation may normally be neglected below 1000°F.

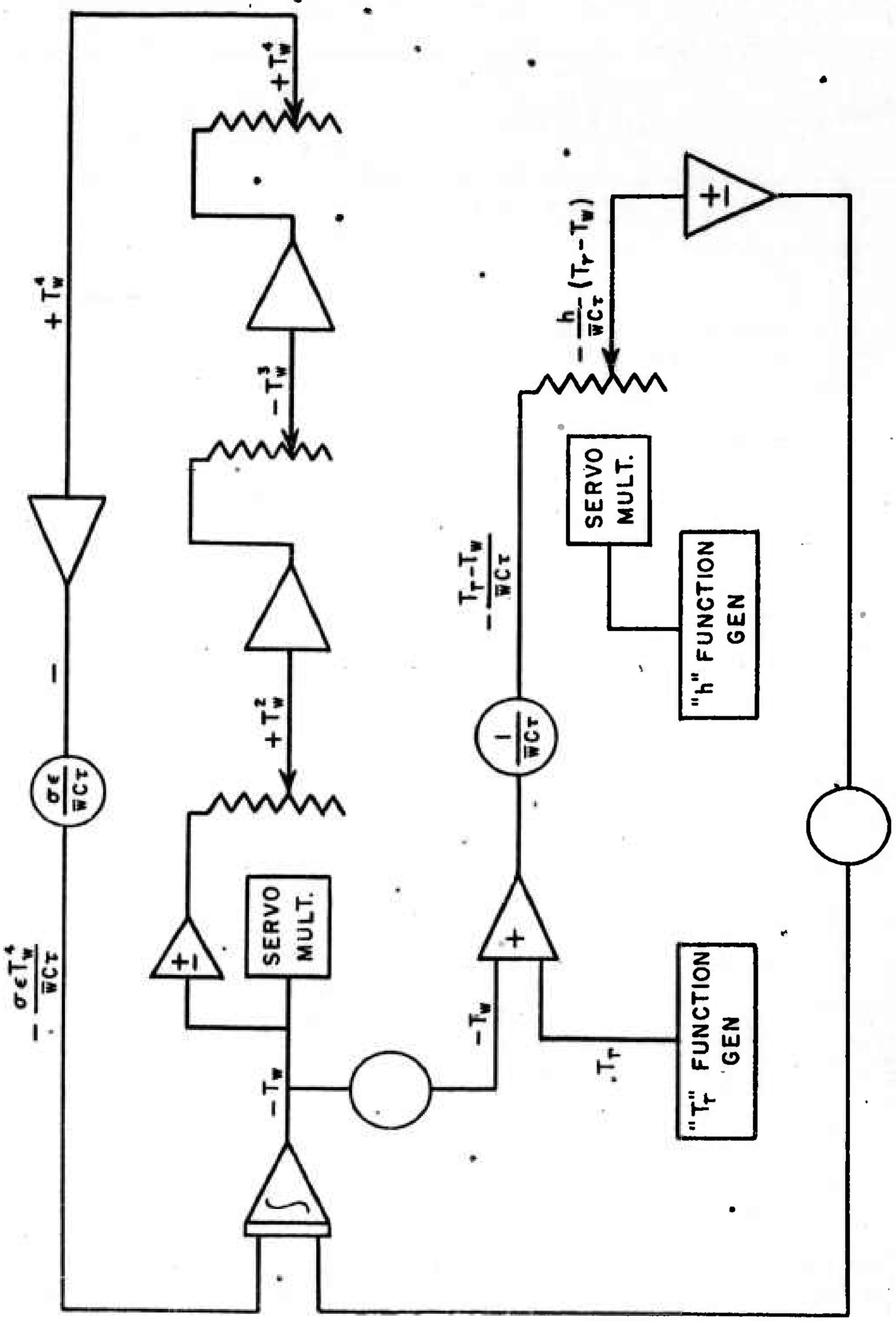

FIG. 10. Analog Computer Schematic for Thermally Thin Skin.

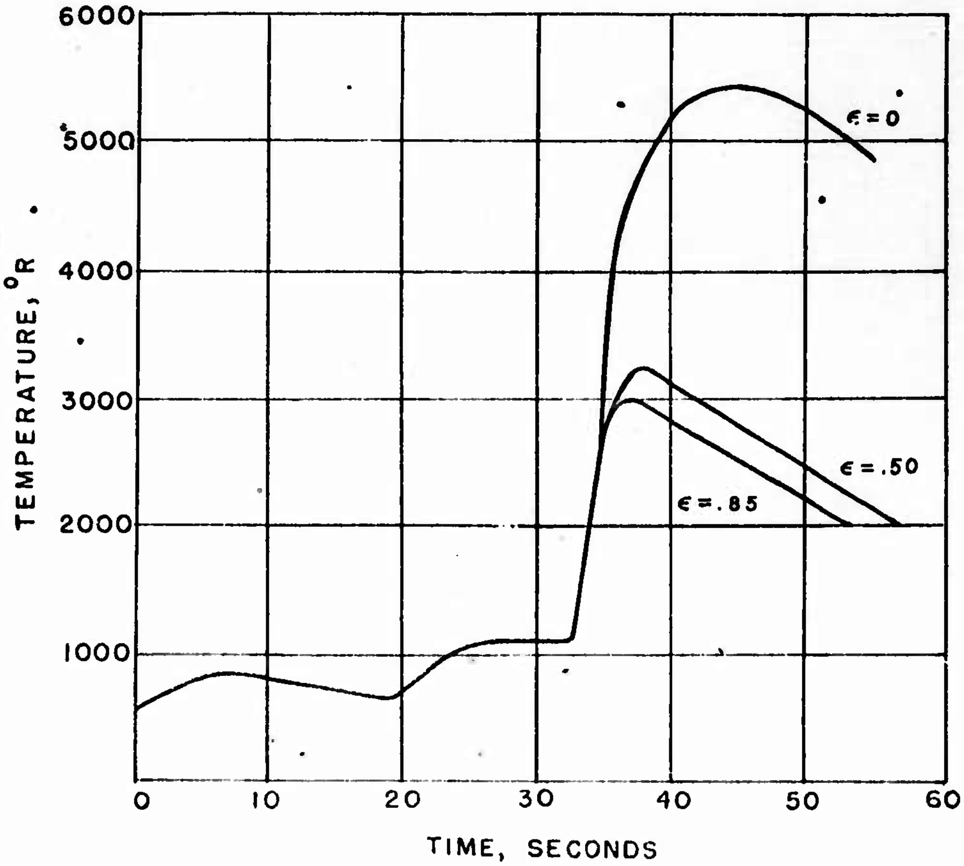

FIG. 11. Wall Temperature Versus Time--Stainless Steel 6.9-inch Diameter Dome, 0.030-inch Thick.

THERMALLY THICK SECTIONS

If the temperature gradient across a particular configuration is greater than 10%, the configuration is termed thermally thick. In thermally thick designs, conduction must be considered to obtain correct solutions. The thermally thick design for this example was an Astrolite hemispherical dome as shown in Fig. 9. As with thermally thin configurations, radiation effects on the outside surface temperature may be neglected when the temperature of this surface is below 1000°F.

The general equation for unsteady-state conduction as given by Fourier's general law of heat conduction can be written:

$$\frac{\partial T}{\partial t} = \frac{K}{\overline{w}C}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{2}$$

For the one-dimensional heat flow which is being considered, the equation reduces to:

$$\frac{\partial T}{\partial t} = \frac{K}{\overline{w}C} \frac{\partial}{\partial x}\left[\frac{\partial T}{\partial x}\right] \tag{3}$$

Partial differential equations such as Eq. 2 and 3 are not directly soluble using analog computers. The solution for wall temperatures of thermally thick sections may be closely approximated using the following method.

Assume that instead of a section with a continuous temperature gradient, a model is devised composed of a number of thin slabs. Each slab has a uniform temperature across its width. To maintain the same total gradient from outside to inside surface as the original section (with continuous gradient), the temperature gradient of the model must take the form of a step function, each step taking place at a slab interface. Using such a model, a heat balance equation may be written for each slab. The equation of the outside surface slab of such a model is:

$$\frac{h}{\overline{w}C\Delta x}(T_r - T_w) - \frac{\sigma \epsilon T_w^{\,4}}{\overline{w}C\Delta x} - \frac{K}{\overline{w}C(\Delta x)^2}(T_w - T_1) = \frac{\Delta T_w}{\Delta t} \tag{4}$$

The heat balance equation for successive slabs is:

$$\frac{dT_1}{dt} = \frac{K}{\overline{w}C(\Delta x)^2}(T_w - 2T_1 - T_2) \tag{5}$$

$$\frac{dT_2}{dt} = \frac{K}{\overline{w}C(\Delta x)^2}(T_1 - 2T_2 - T_3) \tag{6}$$

Equations such as Eq. 5 and 6 can be written for each slab until the inside surface is reached. Since this method is at best an approximation, the effects of radiation and convection are usually neglected due to their small values.

The number of slabs into which a particular configuration is divided depends on the accuracy desired and the size of the computer available. The maximum permissible temperature gradient per slab thickness was set at 10%. Smaller maximum permissible temperature gradient might be chosen if greater accuracy is desired.

The analog computer circuitry needed for computer solution of Eq. 4, 5, and 6 is shown in Fig. 12. This circuit includes solutions for the outside surface and the first two slabs. Additional circuitry may be added to this basic circuit until a model with the correct number of slabs is obtained.

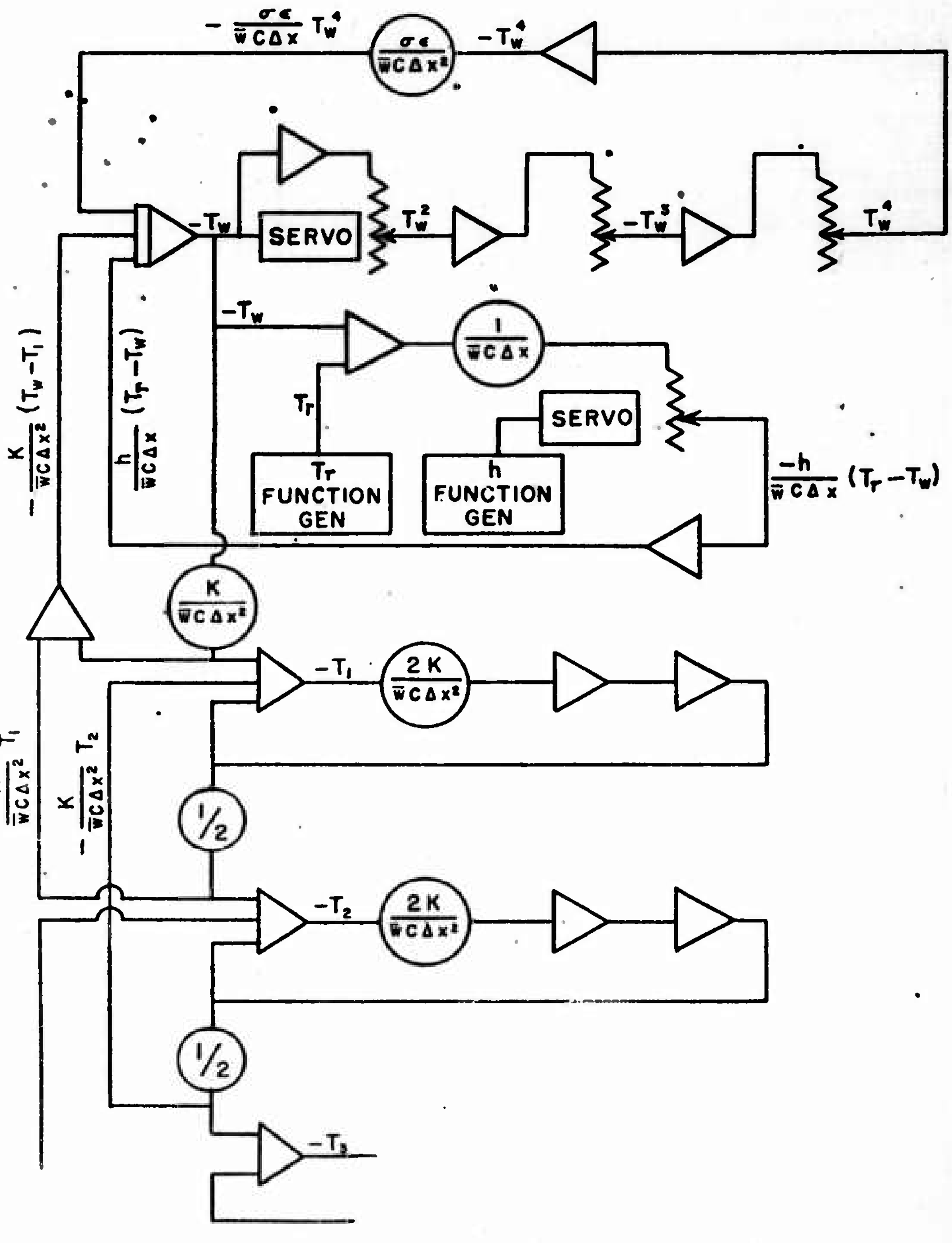

FIG. 12. Analog Computer Schematic for Thermally Thick Skin.

A method has been worked out for obtaining accurate solutions of Eq. 4 using hand calculations (Ref. 7). As with the thermally thin case, however, the method is time consuming and tedious for most applications. Although no provision for radiation effects is included in this method, an approximation of its effect on outside surface temperature may be made by calculating the heat lost due to radiation.

Figures 13, 14, and 15 show some of the solutions obtained. These solutions include the effect of radiation on outside surface temperature (Fig. 13), temperature gradient across the section transient (Fig. 14), and the effect of total thickness on the inside surface temperature (Fig. 15).

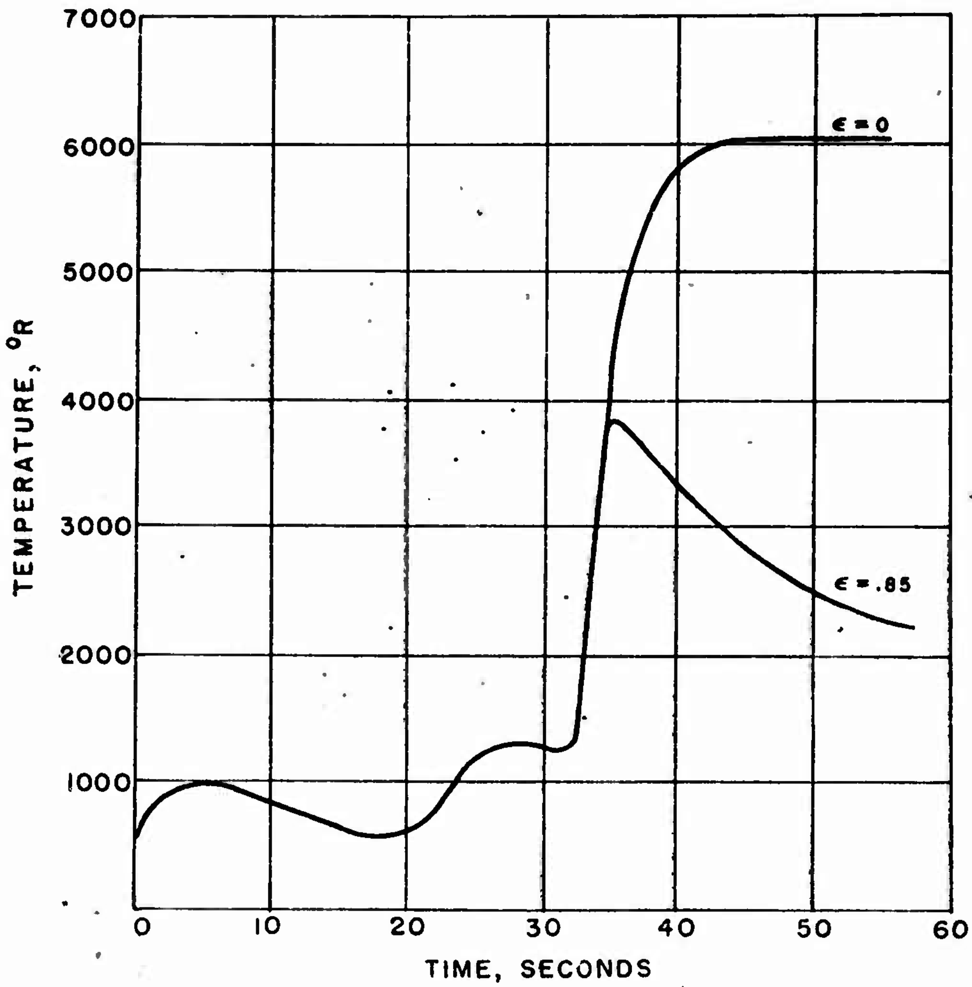

FIG. 13. Outside Wall Temperature Versus Time for Varied Emissivity (ϵ)--Astrolite.

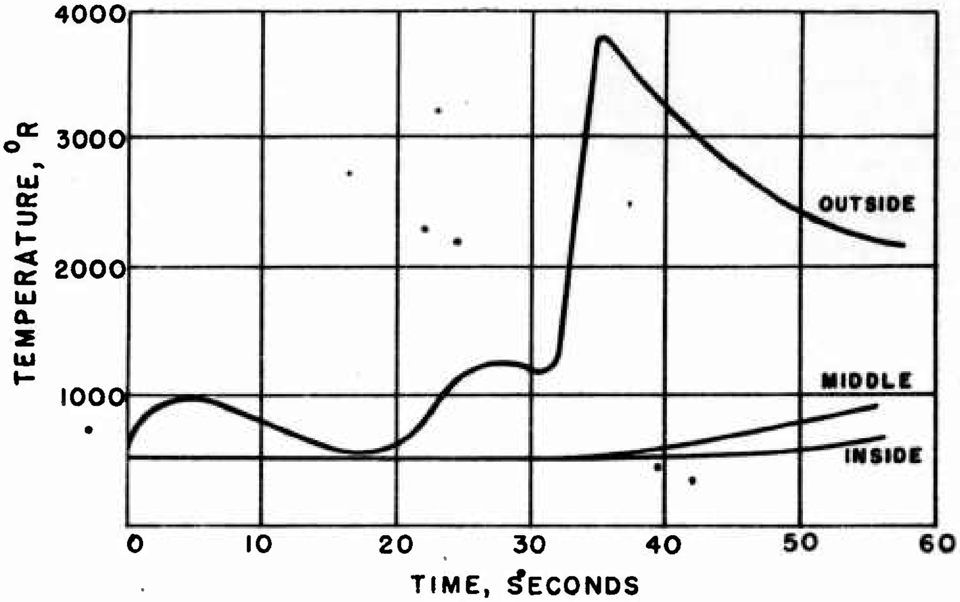

FIG. 14. Wall Temperature Versus Time; Outside, Middle, Inside Astrolite 0.250-inch Thick Wall, ϵ = 0.85

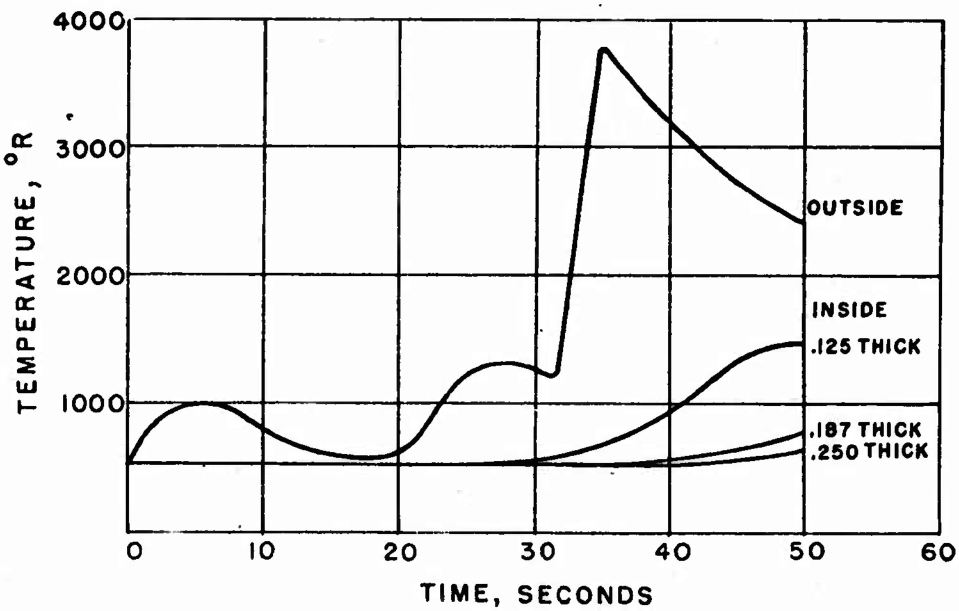

FIG. 15. The Effect of Wall Thickness on Inside Wall Temperature.

COMPARISON

As a check on the accuracy of the computer solutions, the method of Ref. 7 was also used to obtain solutions. These calculations were carried out for time increments of 1, 2, and 3 seconds. Figure 16 shows the results of these calculations and the computer solution at the lower recovery temperatures of the flight profile assumed. The agreement between the various solutions is very good. Figure 17 shows the results obtained at the higher recovery temperatures with radiation not included. The computer solution lies generally between the 2- and 3-sec increment solutions.

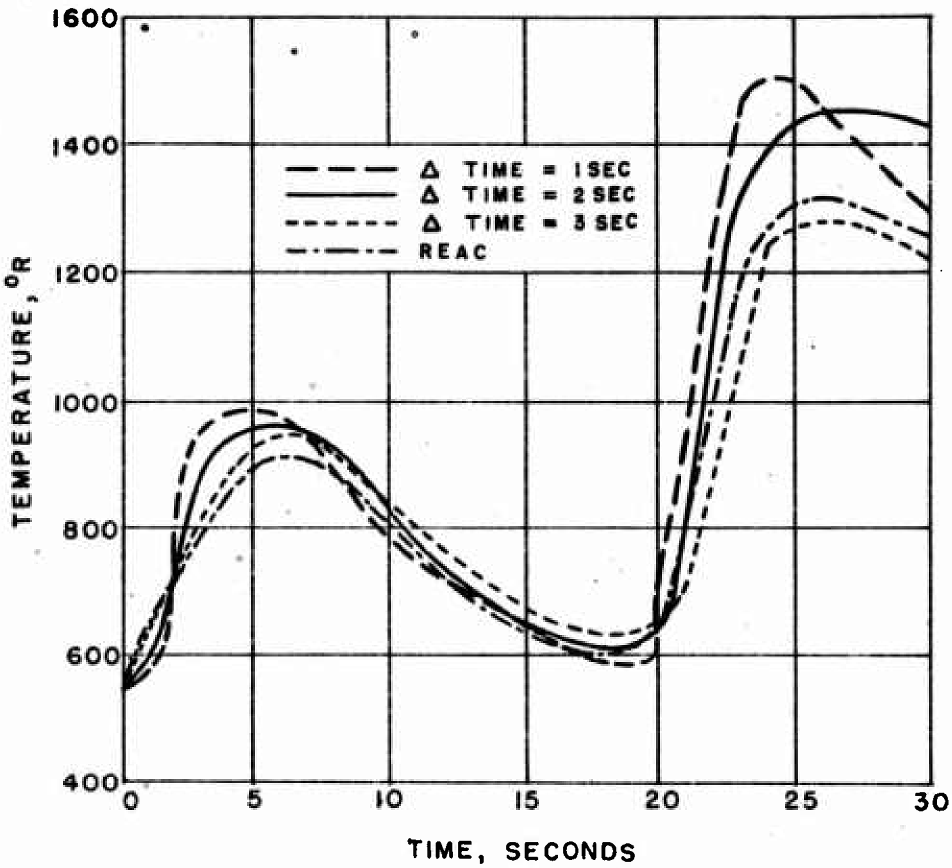

FIG. 16. Temperature Versus Time Comparison.

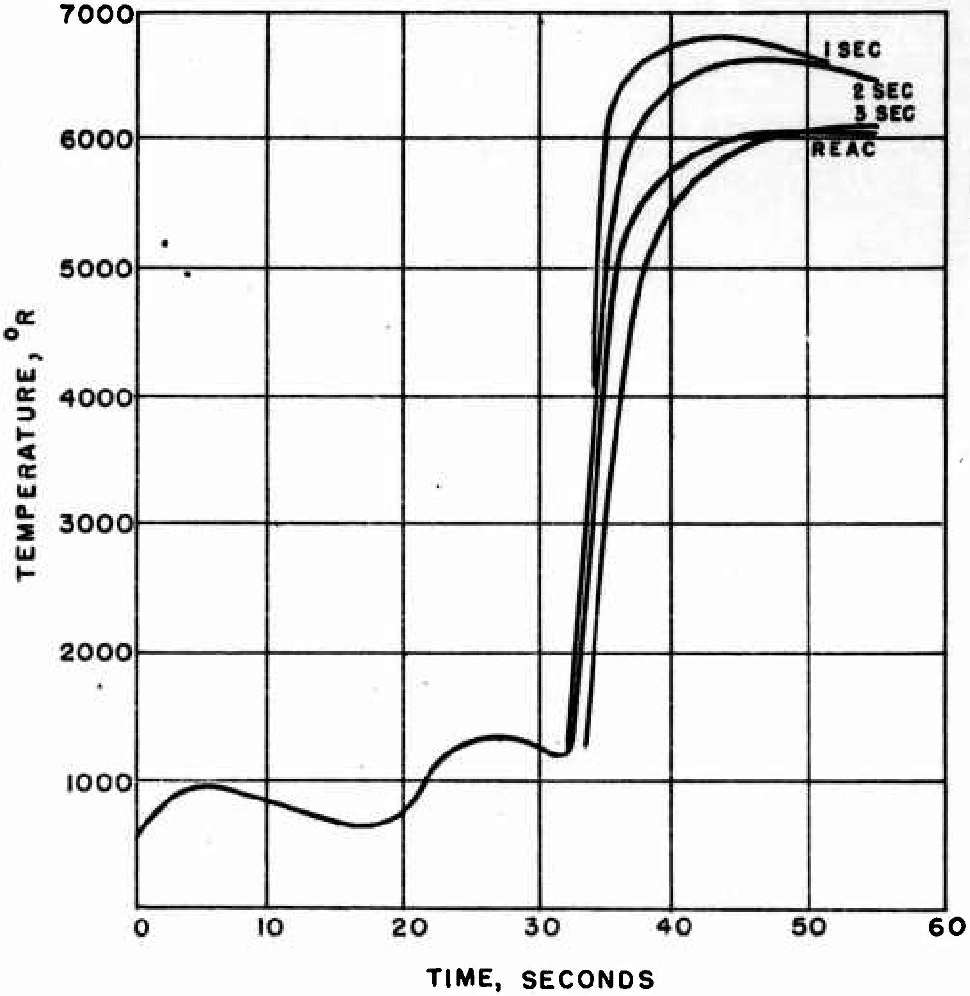

FIG. 17. Outside Wall Temperature Versus Time Comparison--Zero Emissivity, Astrolite.

Figure 18 is a comparison of computer and hand solutions for outside surface temperature. The farter response of the computer solution is effectively shown on this graph.

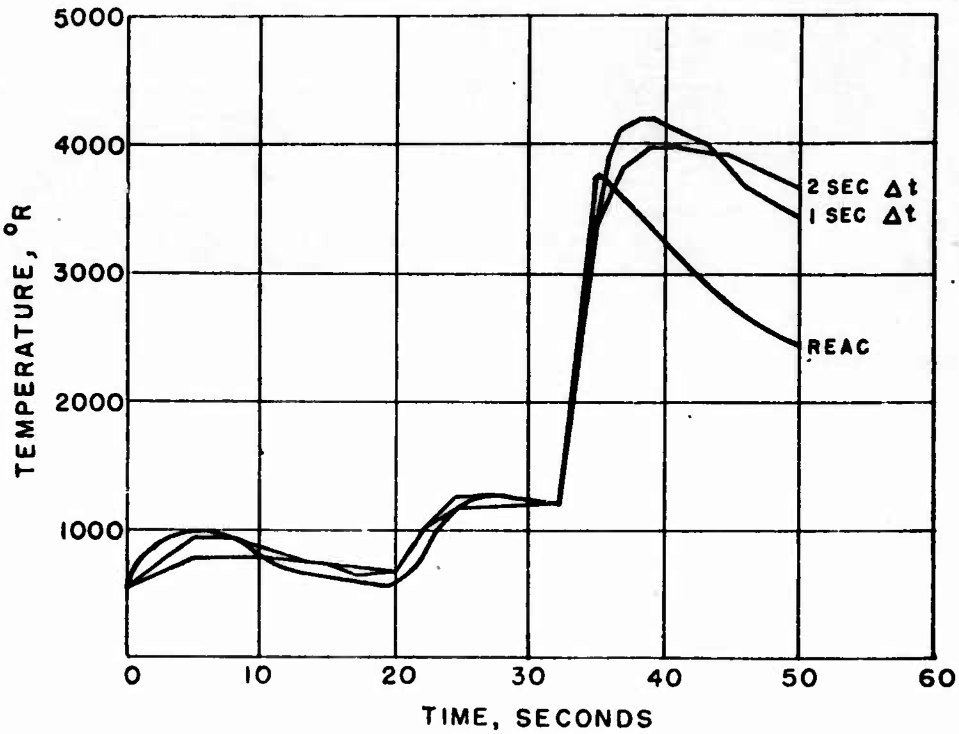

FIG. 18. Hot Wall Temperature Versus Time--Astrolite, 6.9-inch Diameter Dome, 1/4-inch Thick; Emissivity = 0.85.

SUMMARY

The use of an analog computer to obtain solutions of the outside surface temperature of aerodynamically heated bodies results in large time savings over hand solutions of the same problem. After the control panel has been wired, and the recovery temperature and heat transfer coefficient functions have been put into the computer, solutions for a large number of emissivities, wall thicknesses, and even different materials can be obtained in a very short time. These changes are normally accomplished by changing one or more potentiometers in the computer. A completely new set of hand calculations is required, however, for each new set of initial parameters. Also, while radiation effects are included quite simply in computer solutions, they add very much to the complexity of hand solutions.

The accuracy of the computer solution appears to be very good, although further experimental work is needed to show exactly how good.

NOMENCLATURE

C	Specific heat of material, $\frac{Btu}{lb\text{-}°F}$
h	Heat transfer coefficient, $\frac{Btu}{in^2\text{-}sec\text{-}°F}$
K	Thermal conductivity, $\frac{Btu}{in\text{-}sec\text{-}°F}$
T	Absolute temperature, °R
t	Time, seconds
$\dot{w}$	Specific weight, lb/in^3
Δx	Slab thickness, inches
ϵ	Emissivity
σ	Stephan Boltzman constant, $\frac{Btu}{in^2\text{-}sec\text{-}°R^4}$
τ	Total thickness in thermally thin configurations, inches

Subscripts

w	Wall conditions
r	Recovery conditions
n	Slab conditions

SECTION III. WHEN IS A SKIN THERMALLY THIN?

By

Jerome R. Katz

INTRODUCTION

In the problem of aerodynamic heating it has become desirable to know when a layer or skin of an object flying in air, and hence, subjected to heating, can be considered thin. That is, when can the temperature be considered uniform throughout the skin? Thermodynamically, a skin can be considered thin when the material has a high conductivity, or when the time for which it is subjected to heating is long. However, for this report a thin skin will be defined in terms of the thickness of the material, which will be a single layer metal. The skin is approximately thin when the derivative of temperature with respect to the thickness can be considered zero $(\frac{dU}{dx})$; i.e., temperature gradients across its thickness can be neglected. A method for determining a skin thickness is discussed in this section for 1-, 5-, and 10-percent variation in temperature throughout the material. The calculations for heat transfer were performed for a flat plate with heat being added on one side and insulation on the other (Ref. 7). For the purpose of this report, the insulation will simulate a no-heat transfer condition.

METHOD

Reference 7 contains values of transient temperatures which have been normalized. The normalized temperatures are represented by u and are solutions of Eq. 1 of Ref. 7 tabulated for a given Biot modulus, α, x/L ratio, and Fourier modulus, τ. The Fourier modulus, τ, u, and the Biot modulus, α, are defined as follows:

$$\alpha = \frac{hL}{K}$$

$$U = (U_o - U_i)u + U_i \quad , \ (0 \leq u \leq 1)$$

$$kL^2\tau = t, \quad (0.001 \leq \tau \leq 1,000)$$

Since one side was subjected to heat transfer, Eq. 1 can be written:

$$\alpha_o = \frac{h_oL}{K}$$

where L represents plate thickness. The equation representing the conditions on the other side (no heat transfer) is defined by $\alpha_i = 0$.

Each page of the tables in Ref. 7 contains the above data for a different value of α_o ranging from 0.001 to 1,000 thusly: α_o = 0.001, 0.002, 0.004, 0.006, 0.01, 0.02, 0.04, 0.06, 0.1, etc. Hence, this gives 25 pages of data. Now for each page, or each value of α_o, a value of τ can be found corresponding to a 1-, 5-, and 10-percent variation in temperature in the following manner:

First take the ratio $\frac{U_o - U_i}{U_o}$ for each τ entry. By linear interpolation, when the value of the above ratio was found to be 0.01, 0.05, and 0.10, a similar interpolation process was used to yield corresponding values of τ. This gives for each α_o, a τ for which the temperature variation is 1, 5, and 10 percent throughout the skin.

Next, α_o was plotted as a function of τ, $\alpha_o = f_1(\tau)$ with the 1-, 5-, and 10-percent temperature variation as parameters. Now we make use of Eq. 3 and 4 of Ref. 7 and the following relation:

$$k = \frac{c\rho}{K}$$

Combining these expressions yields another function of α_o in terms of τ where L has been eliminated.

$$\alpha_o^2 = \frac{th_o^2}{c\rho K\tau}$$

Let $C = \frac{th_o^2}{c\rho K}$. Then Eq. 5 (Ref. 7) can be written as:

$$\alpha_o^2 = \frac{C}{\tau}$$

For a given C, α_o is a function of τ. Values of C ranging from 0.0001 to 10,000 and τ from 1 to 50 were chosen in order to include a wide range of physical conditions. Then α_o was calculated as a function of τ, $\alpha_o = f_2(\tau)$ on the IBM 704.[2] This data along with the previous $\alpha_o = f_1(\tau)$ curve were plotted in Fig. 20 for the range of C values. The points of intersection of the two functions yield values of α_o that when substituted into the equation

$$\alpha_o = \frac{h_o L}{K}$$

will yield values of L for which the skin of an object of given material can be called thin within the range of values of C and a given percentage of temperature variation.

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[2] The author gratefully acknowledges the assistance of Gladys Radeck for IBM 704 calculations.

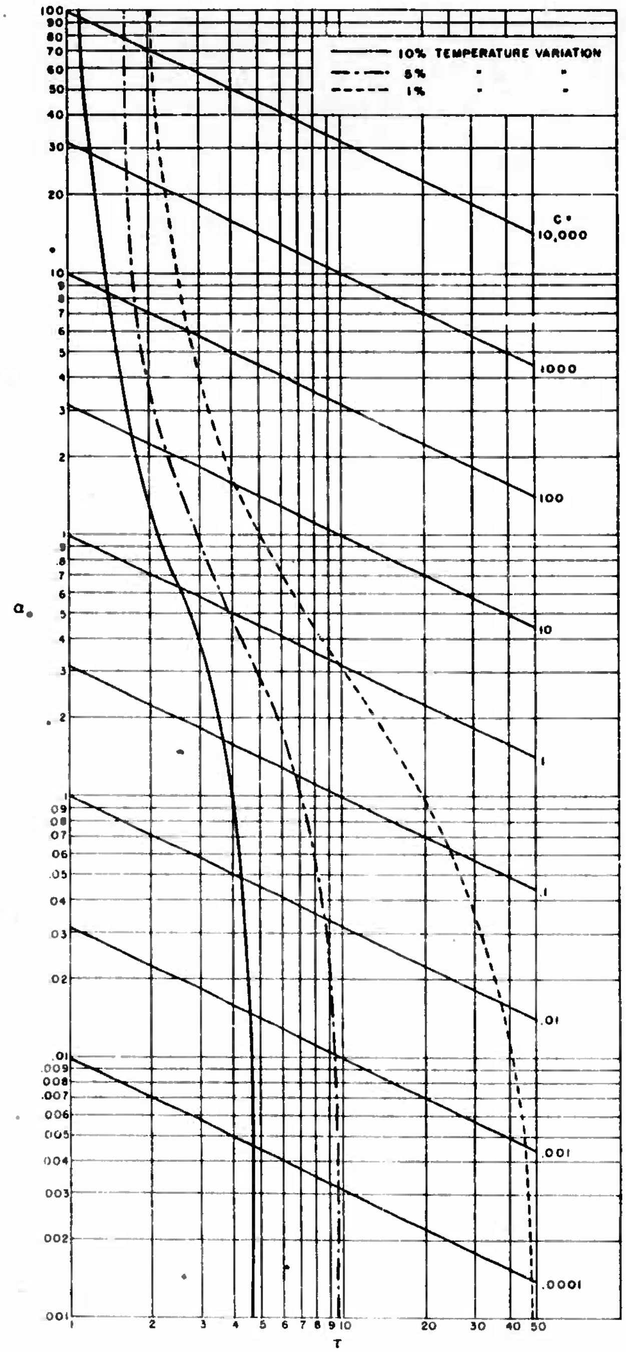

FIG. 19. Biot Modulus as a Function of τ for 1-, 5-, and 10-Percent Temperature Variation in a Flat Plate.

EXAMPLE

To illustrate the method described in the text, a hypothetical problem is selected. It is desired to know for a 1% temperature variation when a flat plate of aluminum alloy can be called thermally thin, after being subjected to heating for 10 seconds. First the physical and thermal properties are found.

$$\rho = 169 \quad lb/ft^3$$

$$K = 117 \quad Btu/hr\text{-}ft\text{-}°F$$

$$c = 0.18 \quad Btu/lb\text{-}°F$$

$$t = 10 \quad sec$$

Selecting a maximum heat transfer coefficient $h_o = 0.01$ Btu/ft^2-sec-°F (Ref. 8), the values of C can be evaluated.

$$C = \frac{t h_o^2}{cK}$$

$$= \frac{(10)(0.01)^2}{(169)(0.18)(117)} \cdot (3600)$$

$$= \frac{3.6}{3560}$$

$$C \cong 0.001$$

Referring to Fig. 20, the line corresponding to $C \doteq 0.001$ is used, and the intersection of this curve with the 1% temperature variation curve is found. This yields an $\alpha_o = 0.0047$. Now using Eq. 1 (Ref. 7), the desired thickness can be obtained.

$$\alpha_o = \frac{h_o L}{K}$$

or

$$L = \frac{\alpha_o K}{h_o}$$

$$= \frac{(0.0047)(0.117)}{(0.01)(3600)} \quad (12)$$

$$L = 0.184 \text{ inches}$$

Approximately the same thickness can be obtained by using an average heat transfer coefficient h_o of 0.003 Btu/ft^2-sec-°F and a time of 100 sec.

NOMENCLATURE

C	$\frac{th_o^2}{c\rho K}$
c	Specific heat of object, Btu/lb-°F
h	Heat transfer coefficient, Btu/ft^2-sec-°F
K	Thermal conductivity, Btu/hr-ft-°F
k	$\frac{1}{\text{thermal diffusivity}} = \frac{1}{\frac{K}{c\rho}} = \frac{c\rho}{K}$
L	Total thickness of skin inches
t	Time, sec
U	Temperature, °F
u	Normalized temperature from tables of Ref. 7
x	Distance through the skin, inches from side exposed to heating
α	Biot modulus based on skin thickness
ρ	Density, lb/ft^3
τ	Fourier modulus, $\frac{t}{kL^2}$

Subscripts

o	Conditions at the exposed side
i	Conditions at the insulated side

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